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Common final exam for Math 117, December 13, 2023.

YOUR NAME: ______________________ SECTION: ______________________

INSTRUCTOR: ______________________

Directions:

- Print your name, section number and your instructor's name on this page in the space provided.
- This exam has 16 questions. Please check that your exam is complete.
- You have two hours to complete this exam. It will be graded out of 68 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator and the list of equations provided by the Department.
- When using decimals round your answers till three decimal places.
- Use of notes, books, any internet resources and electronic devices is NOT allowed.
- You may not communicate with anyone besides the instructor during this exam.

Problem	Score
1	/ 5
2	/ 6
3	/ 2
4	/ 4
5	/ 7
6	/ 4
7	/ 4
8	/ 4
9	/ 5
10	/ 2
11	/ 6
12	/ 5
13	/ 4
14	/ 3
15	/ 4
16	/ 3

Good luck!

1. (Points: 5) A small cafe spends on average \$0.50 to make a cup of coffee. The cafe also has a fixed daily cost of \$450 (for rent, wages, utilities, etc.). Let C, be the cafe's cost, for making x cups of coffee in a day.

   (a) Find a formula for the daily cost C as a function of x cups of coffee. Hint: The cost includes the fixed daily cost as well as the cost for making all cups of coffee sold.

   (b) Find the maximum number of cups of coffee the cafe can make within a budget of \$800.

2. (Points: 6) The weight W, in pounds, of the air in a hot air balloon is a function of H, the temperature of the air in degrees Fahrenheit, and satisfies

$$W = W(H) = \frac{3,969,000}{H + 460}.$$

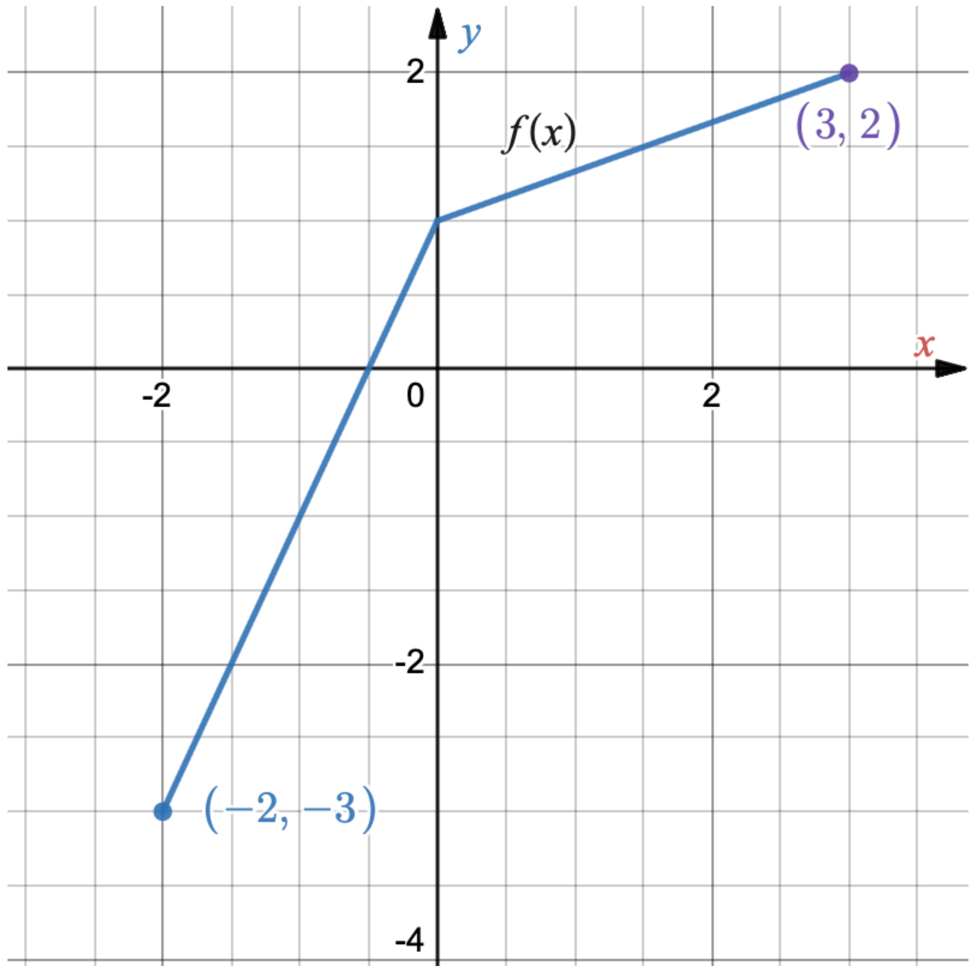
   The force, L, that lifts the balloon, in pounds, is given by

$$L = L(W) = 7489 - W$$

   (a) Find $L(W(225))$.

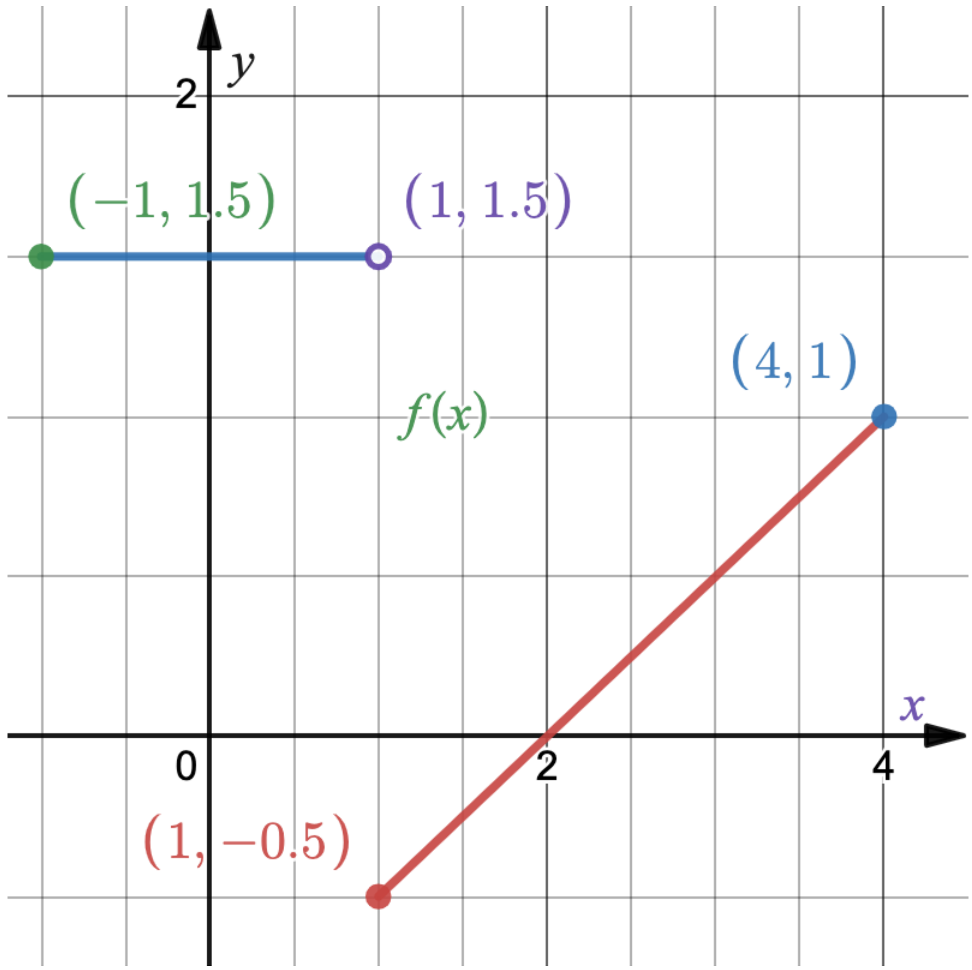
   (b) Explain your answer in one sentence and give the correct units.

3. (Points: 2) Use the complete graph of the function $f(x)$ shown to estimate the domain of the inverse function $f^{-1}(y)$.



4. (Points: 4) Let $f(x) = \dfrac{4-3x}{5x-4}$. Find an expression for $f^{-1}(x)$.

5. (Points: 7) Use the graph of f below to answer the following questions.



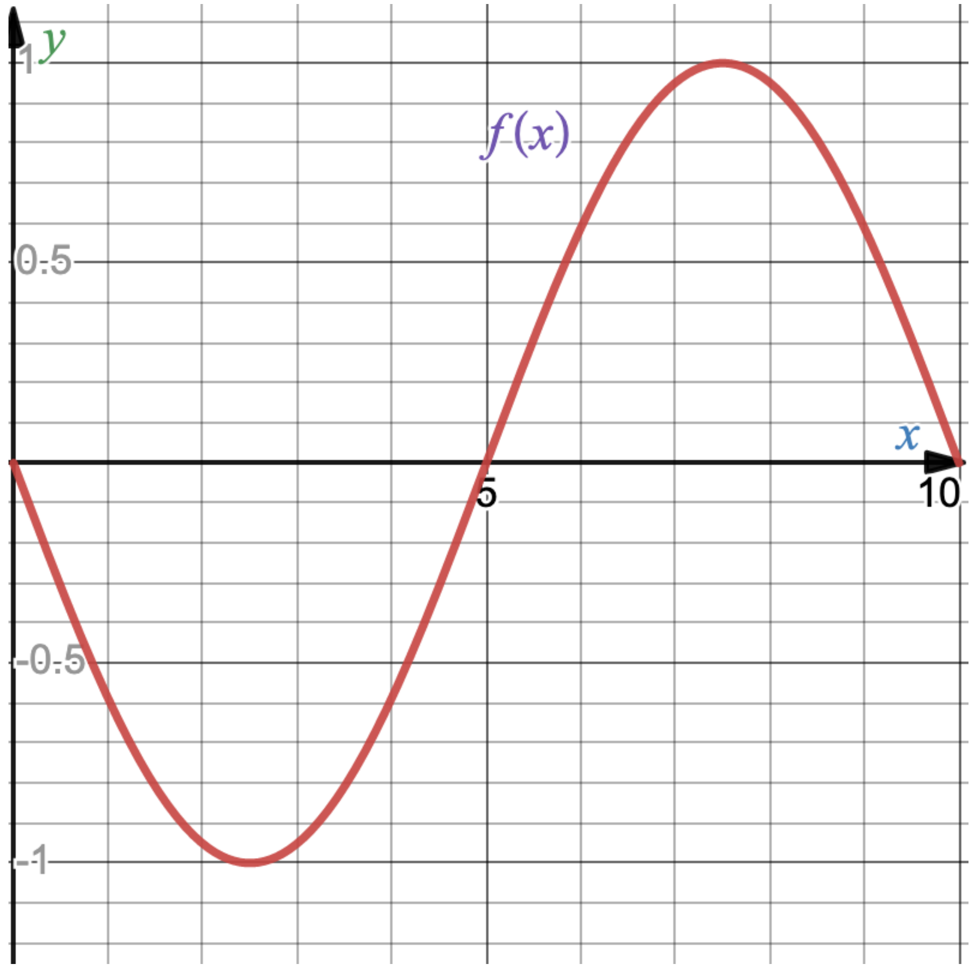
(a) Fill in the blanks to give a piecewise-defined expression for f.

$$f(x) = \begin{cases} ?, & -1 \leq x < 1 \\ ?, & 1 \leq x \leq 4 \end{cases} . \tag{1}$$

(b) Give the domain of f.

(c) Give the range of f.

6. (Points: 4) The graph of $y = f(x)$ is given below.



(a) Give the intervals on which f is simultaneously increasing and concave down.

(b) Give the intervals on which f is simultaneously decreasing and concave up.

7. (Points: 4) The table below gives the atmospheric pressure $P = f(h)$, in torr, at an elevation of h km above the earth's surface.

h (km)	0	2	4	6
P (torr)	760	571	429	323

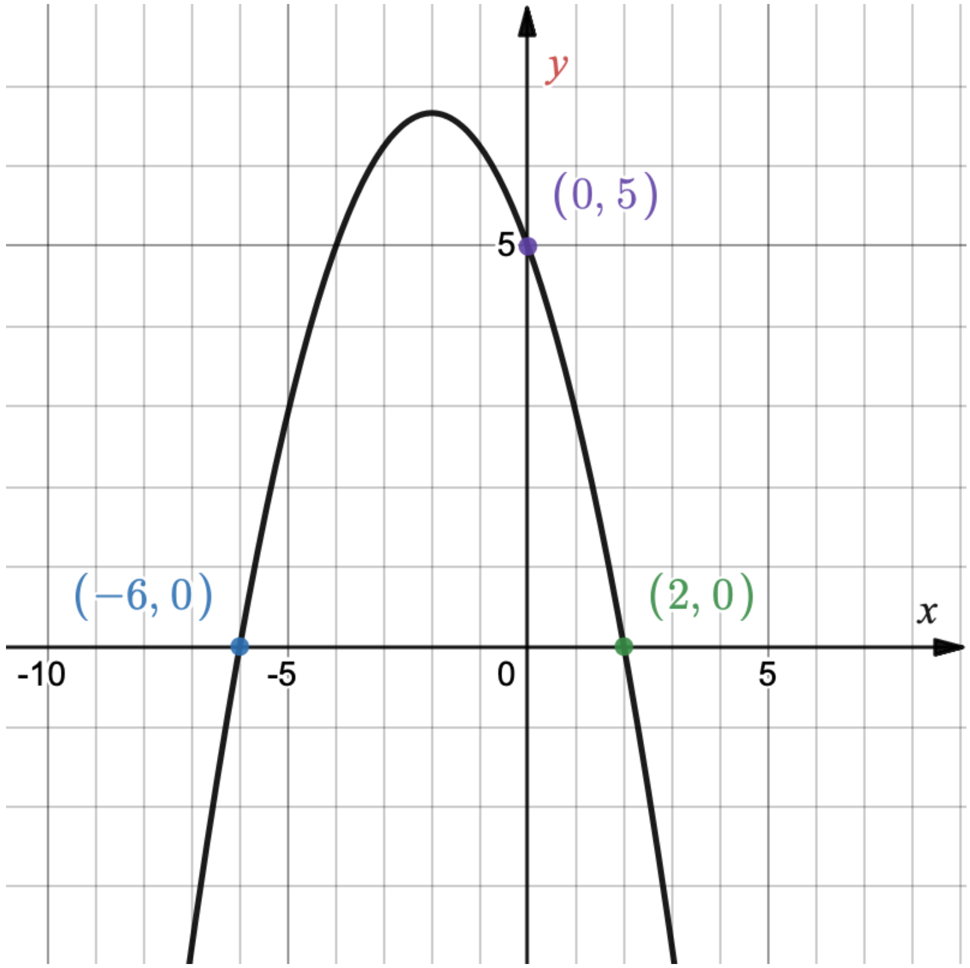
(a) Based on the given information determine if the function $f(h)$ is increasing / decreasing/ neither.

(b) Complete the table below with the average rates of change of f over the given interval.

Interval	$0 \le h \le 2$	$2 \le h \le 4$	$4 \le h \le 6$
Rate of change $\Delta P/\Delta h$	-94.5		

(c) Based on the results of your calculations in the part (b) determine if the function $f(h)$ is concave up / concave down / neither ?

8. (Points: 4) Find a formula for the parabola shown below.

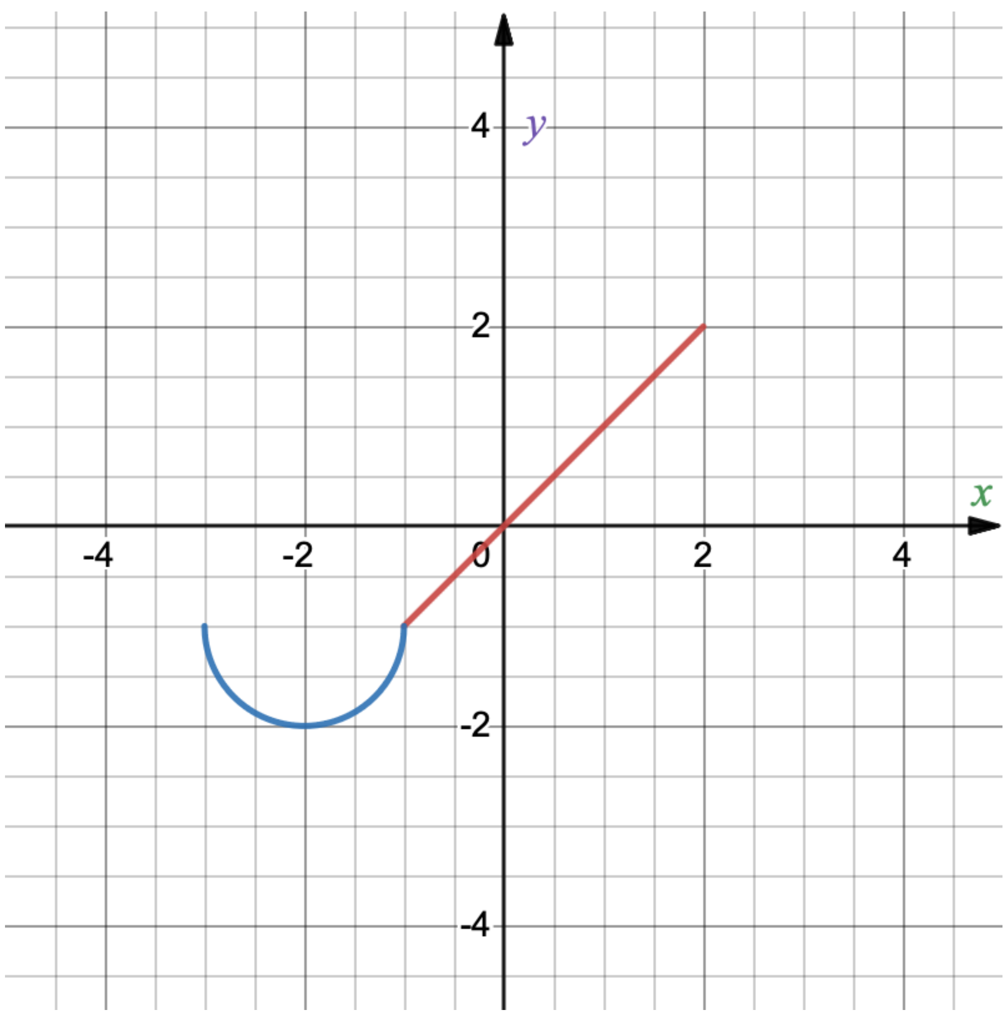


9. (Points: 5) Let $y = 8 - 6x + x^2$.

   (a) Identify the vertex of this function.

   (b) Write the equation in vertex form.

10. (Points: 2) The graph of the function $y = g(x)$, consisting of a line segment and a semicircle is shown for $-3 \leq x \leq 2$. Graph the transformed function $y = g(x-1)+2$ on the same grid.



11. (Points: 6)

   (a) The point $(1, 0)$ is on the graph of $y = p(x)$. Give the coordinates of the corresponding point on the graph of the transformation $q(x) = -p(2x) + 5$.

   (b) Let $p(x) = x^3 - 1$. Write and simplify a formula for the transformation $q(x) = -p(2x) + 5$.

12. (Points: 5)

   The speed of a ship u is related to its hull length l by the following formula

   $$u = k\sqrt{l},$$

   where k is a positive constant.

   (a) Find the constant k if a ship with hull length 225 meters traveling 9 meters/sec?

   (b) Using the formula find the hull length of the ship traveling 6 meters/sec.

13. (Points: 4) Use the function formula $y = x^2(x^2 - 3)(2 + 5x)$ to answer the following questions.

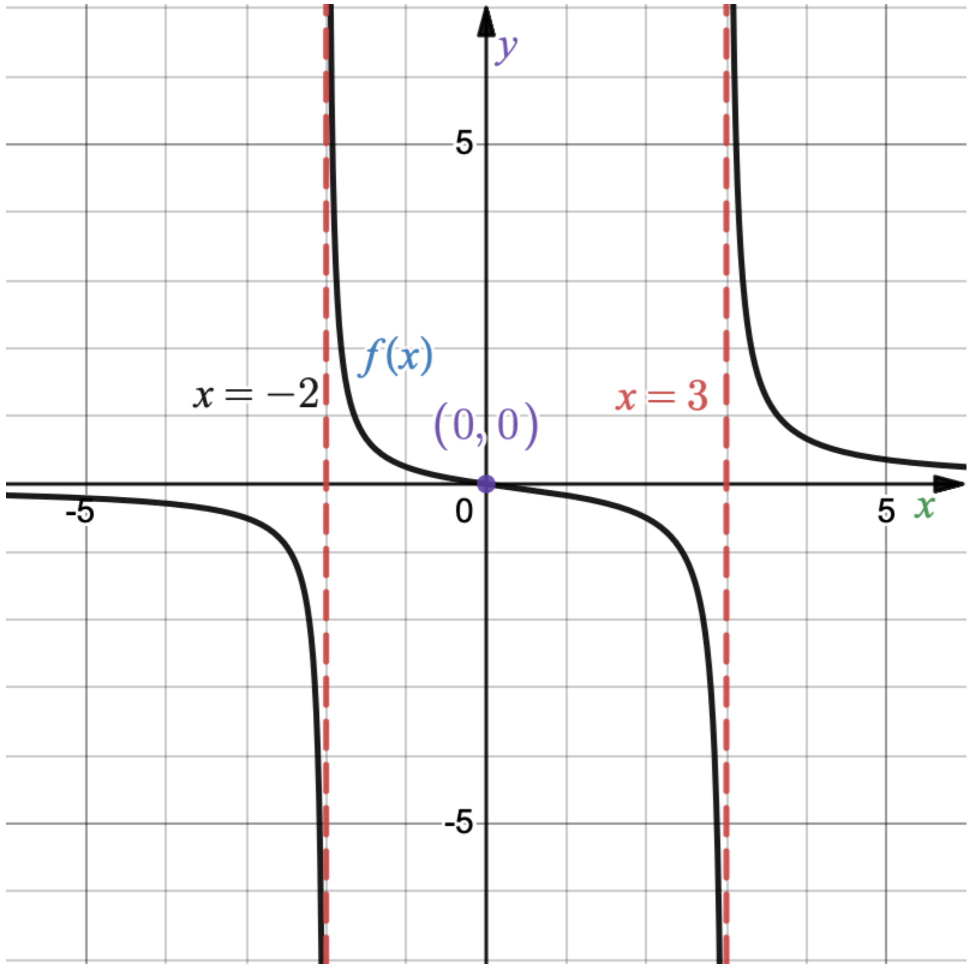
(a) What is the polynomial's leading term?

(b) What is its degree?

(c) Using mathematical notations describe the long-run behavior of the polynomial as $x \to \infty$.

(d) Using mathematical notations describe the long-run behavior of the polynomial as $x \to -\infty$.

14. (Points: 3) The function f is a rational function. Its graph is shown below. Give a possible formula for $f(x)$.



15. (Points: 4) The population of Mathville (in thousand) as a function of time t years after 2000 is given by

$$P(t) = 100\left(\frac{6t+4}{3t+5}\right)$$

(a) Find $\lim_{t\to\infty} P(t)$.

(b) Give a practical interpretation for your result in one sentence.

16. (Points: 3) For the given function $y = \dfrac{x-1}{x^2-4x+3}$ find the coordinates of any holes (if any of those exist).

Formulas

Average rate of change: $\dfrac{f(b)-f(a)}{b-a}$

Slope-intercept form: $y = b + mx$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Quadratic function: $y = ax^2 + bx + c$

Factored form: $y = a(x-r)(x-s)$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vertex form: $y = a(x-h)^2 + k$

Power function $y = kx^p$

Directly proportional: $y = kx$

Inversely proportional: $y = \dfrac{k}{x}$

Factored form of a polynomial: $p(x) = c(x-a_1)(x-a_2)\cdots(x-a_n)$